described in [3]. Here this technique is used for solving the last task. Really, according to descriptions from [1], in the simplex table, potentials are not calculated. Instead of, an objective function is used, i.e., one can write $z = 5x_1 + 7x_2 + 2x_3 + 5x_4$ as $z - 5x_1 - 7x_2 - 2x_3 - 5x_4 = 0.$ "In the simplex table, the objective function is in the last-mentioned form. A variable for which the minimum negative coefficient corresponds in row 0, is introduced into the basis" [1].

Finally, one can note that such tests require less time for solving than usual tasks. As for the practical important, tasks of such type help to reduce the time of solving for students, as well as to evaluate students' ability and knowledge of constructing and solving mathematical models for the teacher.

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## GALAMBOS'S DISCUSSION ON ONE OPEN PROBLEM: BACKGROUNDS AND ANSWERS

In 1869, German mathematician Georg Cantor published the paper [1], where the following expansion of real numbers from $[0,1]$ was introduced:

$$\frac{i_1}{q_1} + \frac{i_2}{q_1 q_2} + \frac{i_3}{q_1 q_2 q_3} + \dots + \frac{i_k}{q_1 q_2 q_3 \dots q_k} + \dots. \qquad (1)$$

Here $(q_k)$ is a fixed sequence of positive integers, $q_k > 1$, and $i_k \in Q_k := \{0, 1, 2, \dots, q_k - 1\}$, as well as $Q = (Q_k)$ is a sequence of alphabets.

Now expansions of form (1) are called *Cantor series*, as well as the existence of such expansion for a fixed number $x \in [0,1]$ is denoted by $\Delta^Q_{i_1 i_2 \dots i_k \dots}$ and is called the representation of $x \in [0,1]$ by a positive Cantor series.

One can remark that in modern mathematics, Cantor series expansions have been intensively studied from different points of view. Such investigations include modelling of various mathematical objects defined in terms of Cantor series and of their generalizations, as well as applications in probability theory, numerical analysis, function theory, etc. Functions with complicated local structure (singular, non-differentiable, or nowhere monotone functions), which have applications in economics and physics are modeled by such expansions. A brief description of such researches is noted in [8].

Considering more detail, one can note (see [8]) investigations of Cantor series such that related to the following topics: various types of the normality of numbers, their relations, and conditions of the existence; the completeness of the Lebesgue measure, as well as the density, topological properties, and the Hausdorff measure of sets whose elements are numbers having the property of the normality of a certain type; a properties of a function of the sum of digits in Cantor expansions; the Hausdorff dimensions of sets whose elements are defined in terms of the frequencies of digits; various statistical properties of real numbers; the conditions under which the family of all possible rank cylinders is faithful for the Hausdorff dimension calculation; properties of sets whose elements have a restriction on using digits in their own representations, etc.

The special attention must be given to the problem on representation of rational numbers by Cantor series. In terms of generalizations of the q-ary numeral system, the problem on expansions of rational numbers is difficult. For the case of Cantor series, G. Cantor, P.A. Diananda, A. Oppenheim, P. Erdös, J. Hančl, E. G. Straus, P. Rucki, R. Tijdeman, P. Kuhapatanakul, V. Laohakosol, D. Marques, Pingzhi Yuan and other scientists studied this problem. However, almost all results are for the case when sequences $(q_k)$ and $(i_k)$ have some restrictions (more information can be found in [8]).

In the research [1], Cantor proven the existence of expansion (1) for all numbers from $[0,1]$ and begun investigations on representations of rational numbers by these series. It is shown that an arbitrary number from $[0,1]$ is a rational number if and only if $(i_k)$ is ultimately periodic under the condition when a sequence $(q_k)$ is periodic. Also, in terms of modify formulations, only the necessity of the following theorem was proven (in the case of positive expansions) with more complicated proofs than in [3]:

Theorem ([1 (partially), 3-5]). A rational number $\frac{p}{r}$ has a finite expansion by a positive or sign-variable Cantor series if and only if there exists a number $n_0$ such that the condition $q_1q_2...q_{n_0} \equiv 0 \pmod{r}$ holds.

In the monograph [2], the problem of representations of rational numbers by Cantor series (1) is called *the fourth open problem*. János Galambos noted the following:

“Problem Four. Give a criterion of rationality for numbers given by a Cantor series. What one should seek here is a directly applicable criterion. A general sufficient condition for rationality would also be of interest, in which the quoted theorems of Diananda and Oppenheim (including the abstract criterion by condensations) can be guides or useful tools. If in a Cantor series, negative and positive terms are permitted, somewhat less is known about rationality or irrationality of the resulting sum. G. Lord (personal communication) tells me that the condensation method can be extended to this case as well, but still, the results are less complete than in the case of ordinary Cantor series." ([4, p. 134]).

Considering the last-mentioned discuss, Cantor series of the following form are considered in [7]:

$$\frac{(-1)^{\rho_1} i_1}{q_1}+\frac{(-1)^{\rho_2} i_2}{q_1 q_2}+\frac{(-1)^{\rho_3} i_3}{q_1 q_2 q_3}+\ldots+\frac{(-1)^{\rho_k} i_k}{q_1 q_2 q_3 \ldots q_k}+\ldots, \tag{2}$$

where $M$ is a fixed subset of the set of all positive integers and

$$\rho_k=\begin{cases}1 \text{ whenever } k \in M \\ 2 \text{ whenever } k \notin M\end{cases}.$$

It is easy to see that the last series are positive Cantor series whenever the condition $k \notin M$ holds for all positive integers $k$.

A useful auxiliary notion is the shift operator, which is defined for positive expansions by the following rule:

$$\sigma(x)=\sigma\left(\Delta^{Q}_{i_1 i_2 \ldots i_k \ldots}\right)=\frac{i_2}{q_2}+\frac{i_3}{q_2 q_3}+\ldots\frac{i_k}{q_2 q_3 \ldots q_k}+\ldots \equiv q_1 \Delta^{Q}_{0 i_2 i_3 \ldots i_k \ldots},$$

$$\sigma^n(x)=\sigma^n\left(\Delta^{Q}_{i_1 i_2 \ldots i_k \ldots}\right)=\frac{i_{n+1}}{q_{n+1}}+\frac{i_{n+2}}{q_{n+1} q_{n+2}}+\ldots\frac{i_{n+k}}{q_{n+1} q_{n+2} \ldots q_{n+k}}+\ldots \equiv q_1 q_2 \ldots q_n \Delta^{Q}_{0 \ldots 0 i_{n+1} i_{3n+2} \ldots i_{n+k} \ldots}.$$

One can note a fact that the shift operator or its compositions map a preimage into the other numeral system. In the other words,

$$\sigma^n:\begin{cases}(i_1, i_2, i_3, \ldots, i_k, \ldots) \to (i_{n+1}, i_{n+2}, i_{n+3}, \ldots) \\ (q_1, q_2, q_3, \ldots, q_k, \ldots) \to (q_{n+1}, q_{n+2}, q_{n+3}, \ldots)\end{cases}.$$

To evaluate digits of representations of rational numbers, this operator plays an important role (see explanations and formulations in [6, 7]).

In [7], investigations are presented in the following topics:

– Expansions by sign variable Cantor series, cylinders, and auxiliary notions of the shift operator which defined in terms of these series;

– General (i.e., in terms of integers sequences $(q_k)$ and $(i_k)$) necessary and suffusion conditions for a rational number to be representable by sign-variable Cantor series.

– Calculating of values of digits in representations of rational numbers.

– The consideration of all items in the mentioned Galambos discussion from [2]. Answers are given on all problem moments.

For positive Cantor series, full investigations about rational numbers are also given in [4-6] including calculating of digits, as well as modelling of rational numbers.

Let us write some more general statement of these result.

Suppose $N$ is the set of all positive integers.

Theorem [7]. *A number $x$ represented by Cantor series is rational for any type of expansion, if and only if there exist numbers $n \in N \cup \{0\}$ and $m \in N$ such that the condition $\sigma^n(x) = \sigma^{n+m}(x)$ holds.*

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## ПРИЙНЯТТЯ УПРАВЛІНСЬКИХ РІШЕНЬ В УМОВАХ РІЗНОЇ ІНФОРМОВАНОСТІ

Проблема вибору управлінських рішень в умовах різної інформованості про стан зовнішнього середовища, тобто сукупності зовнішніх факторів, що впливають на функціонування організації, зводиться до вирішення таких двох питань:

– вибір критерію прийняття рішення;

– визначення стратегії функціонування, що забезпечує найкращу реалізацію вибраного критерію.